Perspectives on the Bayes factor

Jorge N. Tendeiro
Hiroshima University
tendeiro@hiroshima-u.ac.jp

Materials:
https://www.jorgetendeiro.com/talk/2023_unilisboa/

01 March 2023

Outline

The Bayes factor:

1. Introduction.
2. In practice.
3. Properties.
4. In applied research.
5. Conclusions, next steps.

The contents of this talk include materials that I recently presented at a conference:
https://www.jorgetendeiro.com/talk/2023_csp/

Setting

For this talk, I do *not* assume that everyone is...

- ... acquainted with the Bayesian framework.
- ... acquainted with the Bayes factor.
- ... familiar with R nor JASP.

Setting

For this talk, I do *not* assume that everyone is...

- ... acquainted with the Bayesian framework.
- ... acquainted with the Bayes factor.
- ... familiar with R nor JASP.

I included more material than I can discuss in today's talk, *on purpose*.
Those interested should have enough info to follow up afterwards!

1. Bayes factor — Introduction

Bayes factor

Bayes factors are being increasingly advocated as a better alternative to *null hypothesis significance testing* (NHST).[1,2,3,4,5]

[1]Jeffreys (1961) [2]Wagenmakers et al. (2010) [3]Vanpaemel (2010) [4]Masson (2011) [5]Dienes(2014)

Bayes factor — Definition

The Bayes factor[1,2] quantifies the change from prior odds to posterior odds due to the data observed. Consider:

- Two hypotheses (or models) to compare, $\mathcal{H}_0$ *vs* $\mathcal{H}_1$.
- Data D.

Bayes factor — Definition

The Bayes factor[1,2] quantifies the change from prior odds to posterior odds due to the data observed. Consider:

- Two hypotheses (or models) to compare, $\mathcal{H}_0$ *vs* $\mathcal{H}_1$.
- Data D.

Assume that either $\mathcal{H}_0$ or $\mathcal{H}_1$ must hold true.
Then by Bayes' rule ($i = 0, 1$):

$$p(\mathcal{H}_i|D) = \frac{p(\mathcal{H}_i)p(D|\mathcal{H}_i)}{p(\mathcal{H}_0)p(D|\mathcal{H}_0) + p(\mathcal{H}_1)p(D|\mathcal{H}_1)},$$

Bayes factor — Definition

The Bayes factor[1,2] quantifies the change from prior odds to posterior odds due to the data observed.
Consider:

- Two hypotheses (or models) to compare, $\mathcal{H}_0$ *vs* $\mathcal{H}_1$.
- Data D.

Assume that either $\mathcal{H}_0$ or $\mathcal{H}_1$ must hold true.
Then by Bayes' rule ($i = 0, 1$):

$$p(\mathcal{H}_i|D) = \frac{p(\mathcal{H}_i)p(D|\mathcal{H}_i)}{p(\mathcal{H}_0)p(D|\mathcal{H}_0) + p(\mathcal{H}_1)p(D|\mathcal{H}_1)},$$

and dividing member by member leads to

$$\underbrace{\frac{p(\mathcal{H}_0)}{p(\mathcal{H}_1)}}_{\text{prior odds}} \times \underbrace{\frac{p(D|\mathcal{H}_0)}{p(D|\mathcal{H}_1)}}_{\text{Bayes factor, } BF_{01}} = \underbrace{\frac{p(\mathcal{H}_0|D)}{p(\mathcal{H}_1|D)}}_{\text{posterior odds}}.$$

[1] Jeffreys(1939) [2] Kass and Raftery (1995)

Bayes factor — Interpretation (1/2)

$$\boxed{BF_{01} = \frac{p(D|\mathcal{H}_0)}{p(D|\mathcal{H}_1)}}$$

For instance, $BF_{01} = 5$:

> *The data are five times more likely to have occurred under $\mathcal{H}_0$ than under $\mathcal{H}_1$.*

Bayes factor — Interpretation (2/2)

$$\boxed{\underbrace{\frac{p(\mathcal{H}_0)}{p(\mathcal{H}_1)}}_{\text{prior odds}} \times \underbrace{\frac{p(D|\mathcal{H}_0)}{p(D|\mathcal{H}_1)}}_{\text{Bayes factor, } BF_{01}} = \underbrace{\frac{p(\mathcal{H}_0|D)}{p(\mathcal{H}_1|D)}}_{\text{posterior odds}}}$$

For instance, $BF_{01} = 5$:

> *After observing the data, my relative belief in $\mathcal{H}_0$ over $\mathcal{H}_1$ increased by 5 times.*

Bayes factor — Interpretation (2/2)

$$\boxed{\underbrace{\frac{p(\mathcal{H}_0)}{p(\mathcal{H}_1)}}_{\text{prior odds}} \times \underbrace{\frac{p(D|\mathcal{H}_0)}{p(D|\mathcal{H}_1)}}_{\text{Bayes factor, } BF_{01}} = \underbrace{\frac{p(\mathcal{H}_0|D)}{p(\mathcal{H}_1|D)}}_{\text{posterior odds}}}$$

For instance, $BF_{01} = 5$:

> *After observing the data, my relative belief in $\mathcal{H}_0$ over $\mathcal{H}_1$ increased by 5 times.*

This holds regardless of the initial relative belief (i.e., prior odds) of a rational agent.

Prior belief in...		**Prior odds**	$\mathbf{BF}_{01}$	**Posterior odds**	**Posterior belief on...**	
$\mathcal{H}_0$	$\mathcal{H}_1$				$\mathcal{H}_0$	$\mathcal{H}_1$
$1/2 = .50$	$1/2 = .50$	1	5	5	$5/6 = .83$	$1/6 = .17$
$2/3 = .67$	$1/3 = .33$	2	5	10	$10/11 = .91$	$1/11 = .09$
$1/10 = .01$	$9/10 = .90$	$1/9$	5	$5/9$	$5/14 = .36$	$9/14 = .64$

Bayes factor — Possible values

$BF_{01} = \frac{p(D|\mathcal{H}_0)}{p(D|\mathcal{H}_1)} \in [0, \infty)$:

- $BF_{01} > 1 \longrightarrow$ Evidence in favor of $\mathcal{H}_0$ over $\mathcal{H}_1$.
- $BF_{01} = 1 \longrightarrow$ Equal support for either model.
- $BF_{01} < 1 \longrightarrow$ Evidence in favor of $\mathcal{H}_1$ over $\mathcal{H}_0$.

Bayes factor — Possible values

$BF_{01} = \frac{p(D|\mathcal{H}_0)}{p(D|\mathcal{H}_1)} \in [0, \infty)$:

- $BF_{01} > 1 \longrightarrow$ Evidence in favor of $\mathcal{H}_0$ over $\mathcal{H}_1$.
- $BF_{01} = 1 \longrightarrow$ Equal support for either model.
- $BF_{01} < 1 \longrightarrow$ Evidence in favor of $\mathcal{H}_1$ over $\mathcal{H}_0$.

Some qualitative cutoff labels have been suggested, for instance[1,2,3].

Here's Kass and Raftery's classifier:

$\mathbf{BF}_{01}$	**Strength of evidence in favor of $\mathcal{H}_0$**
1 – 3	Not worth more than a bare mention
3 – 20	Positive
20 – 150	Strong
$>$ 150	Very strong

For $BF_{01} < 1$, use $BF_{10} = \frac{1}{BF_{01}}$ as strength of evidence in favor of $\mathcal{H}_1$.

[1] Jeffreys (1939) [2] Kass and Raftery (1995) [3] Lee and Wagenmakers (2013)

Bayes factor — Computation

$$\boxed{BF_{01} = \frac{p(D|\mathcal{H}_0)}{p(D|\mathcal{H}_1)}}$$

Bayes factor — Computation

$$\boxed{BF_{01} = \frac{p(D|\mathcal{H}_0)}{p(D|\mathcal{H}_1)}}$$

Essentially, any two statistical models that make predictions are in theory eligible to be compared via the Bayes factor.

We "just" need to evaluate each model's marginal likelihood:

$$P(D|\mathcal{H}_i) = \int_{\Theta_i} \underbrace{p(D|\theta, \mathcal{H}_i)}_{\text{likelihood}} \underbrace{p(\theta|\mathcal{H}_i)}_{\text{prior}} d\theta.$$

Bayes factor — Computation

$$\boxed{BF_{01} = \frac{p(D|\mathcal{H}_0)}{p(D|\mathcal{H}_1)}}$$

Essentially, any two statistical models that make predictions are in theory eligible to be compared via the Bayes factor.

We "just" need to evaluate each model's marginal likelihood:

$$P(D|\mathcal{H}_i) = \int_{\Theta_i} \underbrace{p(D|\theta, \mathcal{H}_i)}_{\text{likelihood}} \underbrace{p(\theta|\mathcal{H}_i)}_{\text{prior}} d\theta.$$

There are various numerical procedures for this.[1,2,3,4,5,6,7,8]
As of recently, bridge sampling[7] has been of great practical use (in combination JAGS, Stan, or NIMBLE).

[1] Berger and Pericchi (2001) [2] Carlin and Chib (1995) [3] Chen, Shao, and Ibrahim (2000) [4] Gamerman and Lopes (2006)
[5] Gelman and Meng (1998) [6] Green (1995) [7] Gronau et al. (2017) [8] Kass and Raftery (1995)

Bayes factor — Computation

$$\boxed{BF_{01} = \frac{p(D|\mathcal{H}_0)}{p(D|\mathcal{H}_1)}}$$

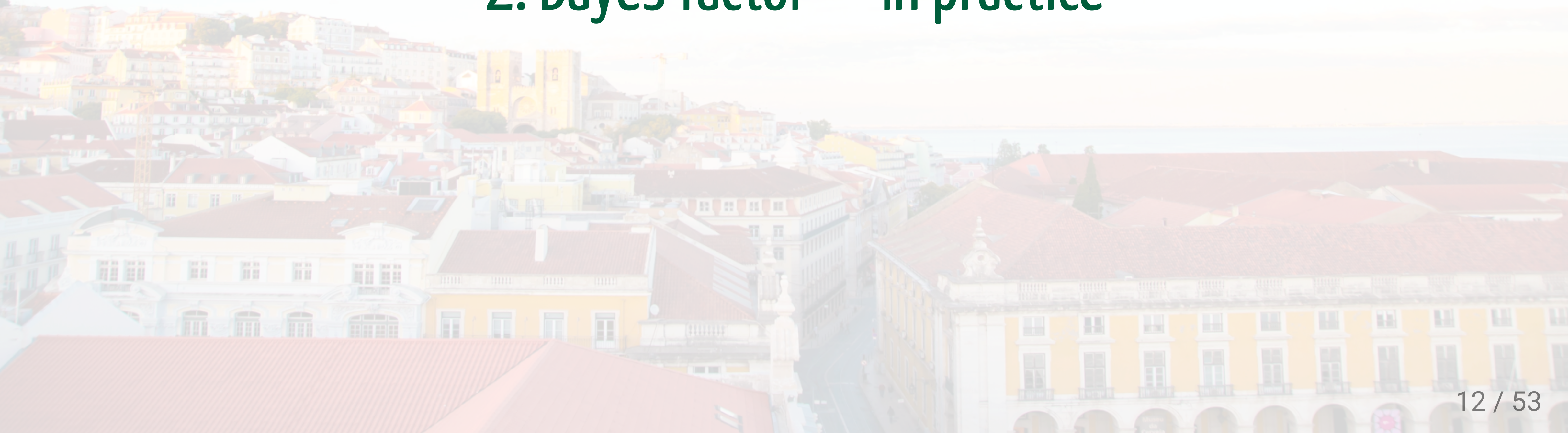
For simpler models there are a few R packages available to assist with the computations:

- `BayesFactor`[1] (mostly used).
- `bain`[2].
- `easystats`[3].
- `bayestestR`[4].
- `brms`[5] and `rstanarm`[6], relying on the `bridgesampling`[7] package.

There is also JASP, a handy and open source GUI.

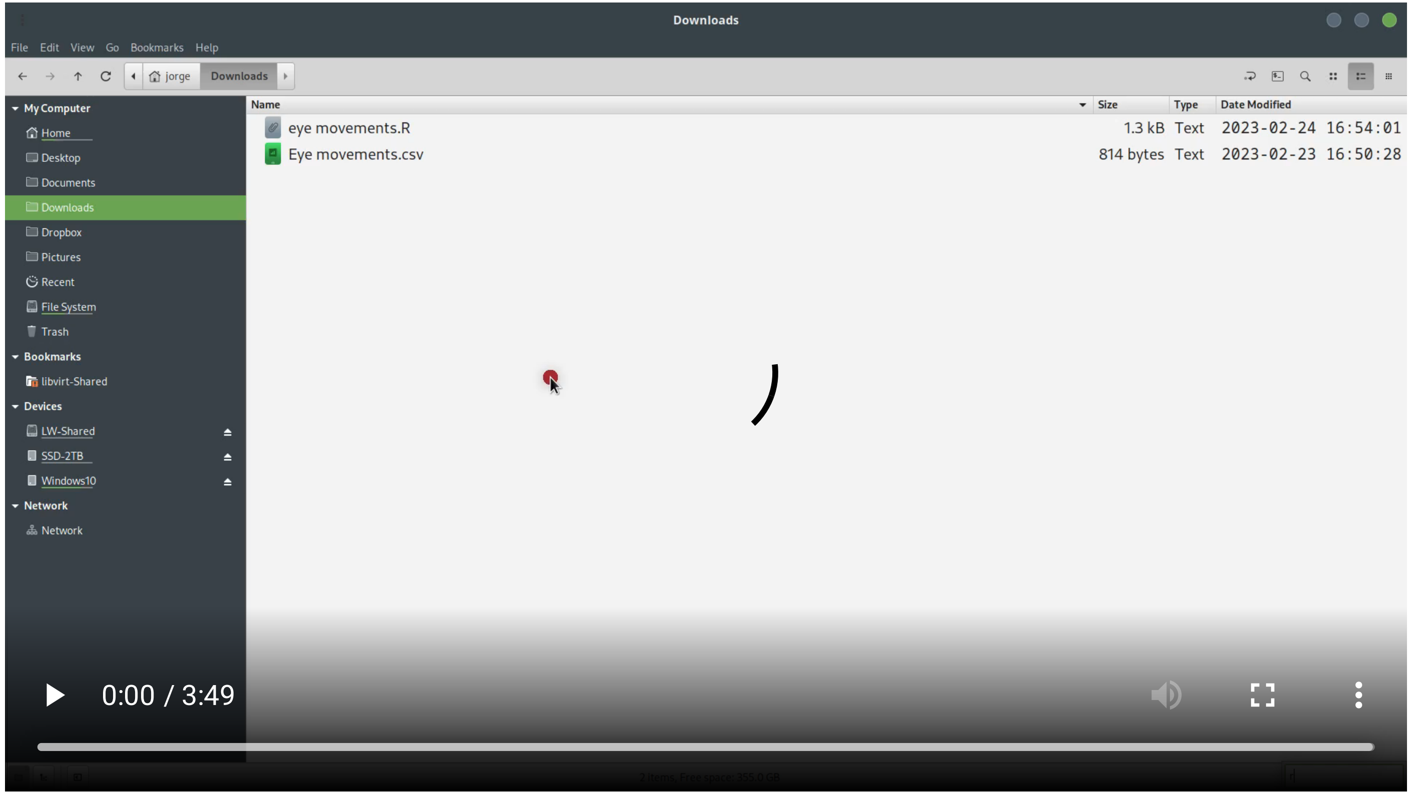
[1]Morey and Rouder (2022) [2]Gu et al. (2021) [3]Lüdecke et al. (2022) [4]Makowski, Ben-Shachar, and Lüdecke (2019)
[5]Bürkner (2021) [6]Goodrich et al. (2022) [7]Gronau, Singmann, and Wagenmakers (2020)

2. Bayes factor — In practice

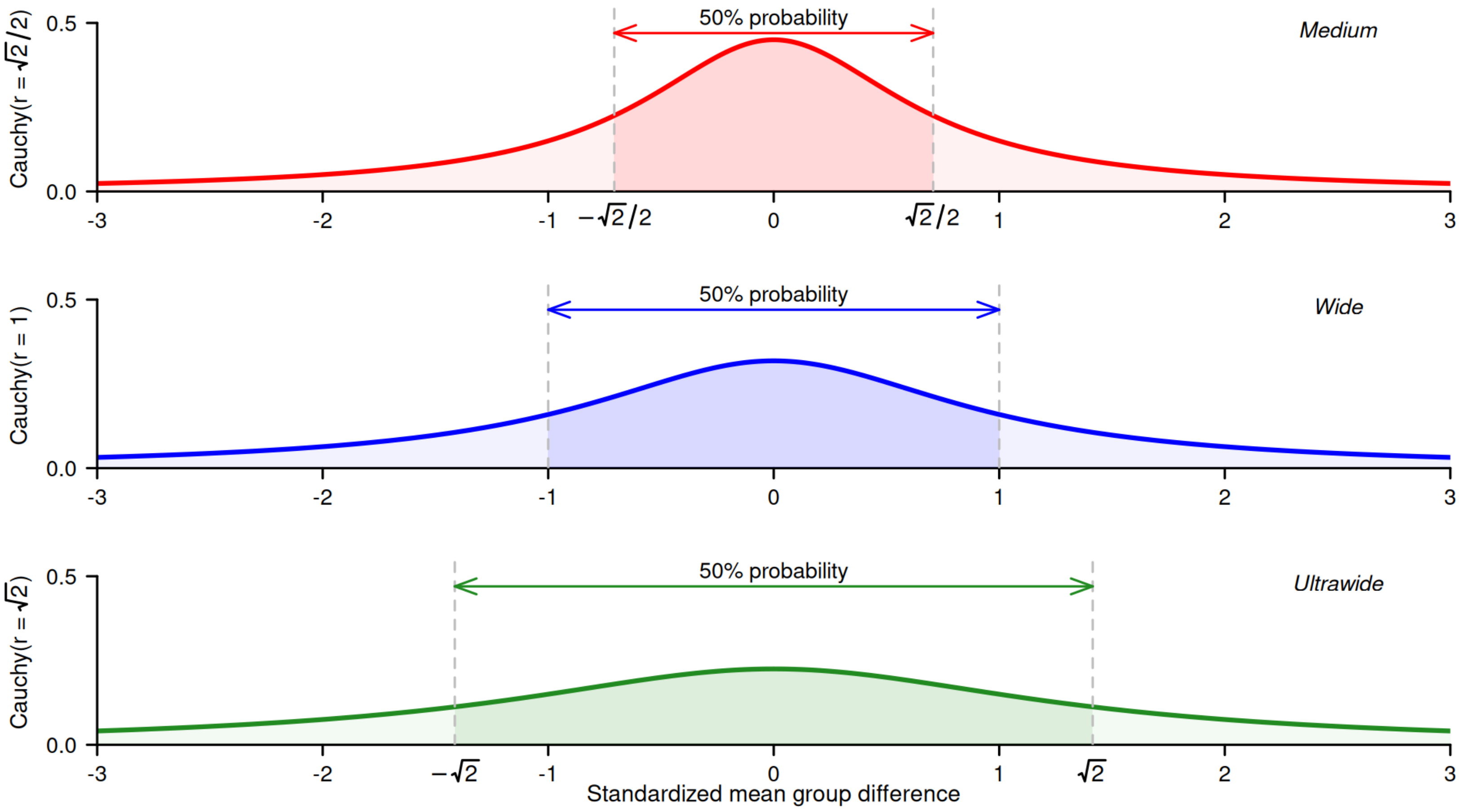


Bayes factor — In JASP

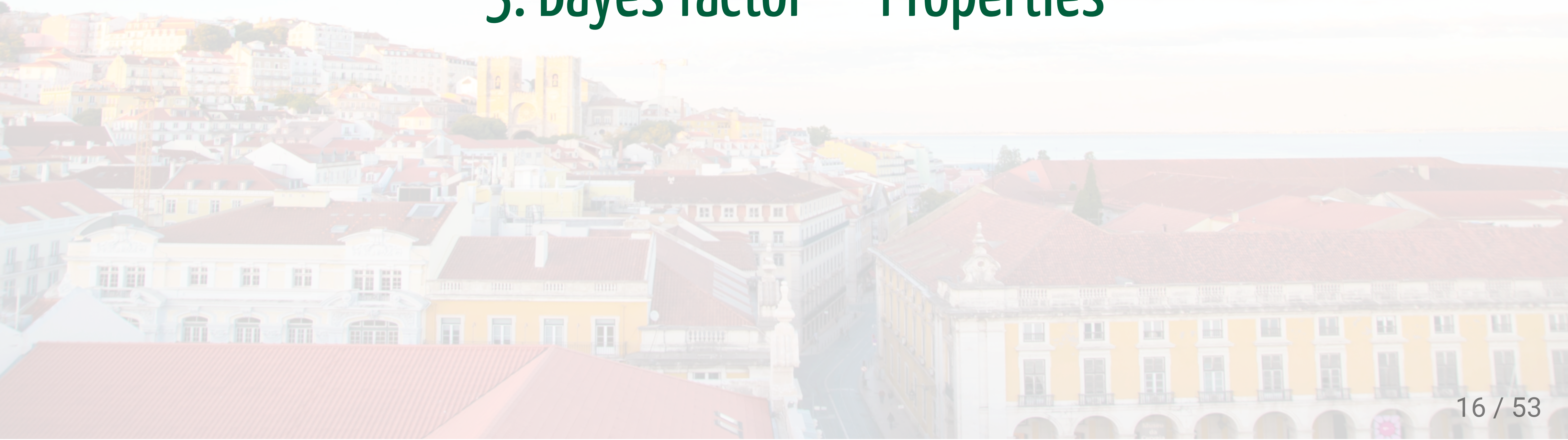
Bayes factor — In R



Bayes factor — Default priors



3. Bayes factor — Properties



Bayes factor — Critical appraisal

Bayes factor have been praised in many instances.[1,2,3,4,5]

But, surprisingly, I could not find many sources with critical appraisals of the Bayes factor.

Bayes factor — Critical appraisal

Bayes factor have been praised in many instances.[1,2,3,4,5]

But, surprisingly, I could not find many sources with critical appraisals of the Bayes factor.

I have been doing this for a few years now.[6,7,8,9]

[1] Dienes (2011) [2] Dienes (2014) [3] Masson (2011) [4] Vanpaemel (2010) [5] Wagenmakers et al. (2018) [6] Tendeiro and Kiers (2019)
[7] Tendeiro, Kiers, and Ravenzwaaij (2022) [8] Tendeiro and Kiers (2023a) [9] Tendeiro and Kiers (2023b)

Bayes factor — Some properties

- Bayes factors are not posterior odds!
- Bayes factors are (at least *can be*) sensitive to priors!
- Bayes factors are a measure of relative evidence!
- Bayes factors can not establish absence/presence!
- Bayes factors are not an effect size measure!
- Inconclusive evidence is not evidence of absence!
- Bayes factors are a continuous measure of relative evidence!

Bayes factor — Some properties

For the rest of this presentation, I will:

- Present the results of a study aiming at studying the occurrence of misconceptions in the literature.
- Explain each misconception.
- Speculate on why these misconceptions come about.

4. Bayes factors — In applied research

Bayes factors — In applied research

Until recently, there was no characterization of the use of the Bayes factor in applied research.
Wong and colleagues[1] were the first to start unveiling the current state of affairs.

Bayes factors — In applied research

Until recently, there was no characterization of the use of the Bayes factor in applied research.
Wong and colleagues[1] were the first to start unveiling the current state of affairs.

In an ongoing effort, I am currently extending the work of Wong et al..
Here I report the details and main findings of my study.
Work with Henk Kiers, Rink Hoekstra, Tsz Keung Wong, and Richard Morey.

Preprint (under review):
https://psyarxiv.com/du3fc/

[1] Wong, Kiers, and Tendeiro (2022)

Context

Background
Social Sciences.

Target:
NHBT and the Bayes factor in particular.

Motivation:
Bayes factors have been regularly used since, say, 2010.
It is very recent.
Not many researchers have received formal training.
It is unclear how things are working out.

Advanced literature search

Google Scholar (2010–):

> ```
> ("bayes factor" AND "bayesian test" AND psychol)
> ```

Web of Science:

> ```
> (TI=((bayes factor OR bayes* selection OR bayes* test*) AND psycho*) OR
> AB=((bayes factor OR bayes* selection OR bayes* test* OR bf*) AND psychol*) OR
> AK=((bayes factor OR bayes* selection OR bayes* test* OR bf*) AND psychol*))
> AND PY=(2010-2022)
> ```

$109 + 58 = 167$ papers (after selection).

Grading criteria

	Criterion	Brief description
QRIP	1 – Describing the BF as posterior odds	Defining or elaborating on BFs as posterior odds ratios.
	3a – Missing explanation for the chosen priors	The reason or justification for the chosen priors is not provided.
	3b – No mention to the priors used	It is unclear which priors were used under either model.
	3c – Incomplete info regarding the priors used	E.g., only providing the distribution family ("Cauchy").
	4 – Not referring to the comparison of models	Presenting BFs as absolute evidence for one of the two models.
	5 – Making absolute statements	Based on the BF, concluding that there is (not) an effect.
	6 – Using BF as posterior odds	Interpreting BFs as ratios of posterior model probabilities.
	7 – Considering BF as effect size	Associating the size of the BF to the size of the effect.
	9 – Inconclusive evidence as evidence of absence	Stating that there is no effect when faced with inconclusive evidence.
	10 – Interpreting ranges of BF values only	Interpreting the Bayes factor simply using cutoffs (e.g., 1-3, 3-10).
Usage	A – Default prior	Justifying using a prior because it is 'the' default.
	B – Null results	Bayes factors as a follow-up to non-significant outcomes from NHST.
	C – Presence *versus* absence	Bayes factors to distinguish between the presence and the absence of an effect.

Results

	Criterion	Count	Percentage
QRIP	1 – Describing the BF as posterior odds	22	13.2%
	3a – Missing explanation for the chosen priors	18	10.8%
	3b – No mention to the priors used	50	29.9%
	3c – Incomplete info regarding the priors used	10	6.0%
	4 – Not referring to the comparison of models	104	62.3%
	5 – Making absolute statements	59	35.3%
	6 – Using BF as posterior odds	34	20.4%
	7 – Considering BF as effect size	7	4.2%
	9 – Inconclusive evidence as evidence of absence	6	3.6%
	10 – Interpreting ranges of BF values only	9	5.4%
Usage	A – Default prior	59	35.3%
	B – Null results	27	16.2%
	C – Presence *versus* absence	30	18.0%

Results

Overall:

- 149 papers (89.2%) displayed at least one QRIP.
- 104 papers (62.3%) displayed at least two QRIPs.

Discussion of the results

We reasoned over the reasons behind the found problems.

Below is a selected synopsis of our considerations.

4. Bayes factors — In applied research

Bayes factors are *not* posterior odds

Bayes factors are *not* posterior odds — *Explanation*

$$\underbrace{\frac{p(\mathcal{H}_0)}{p(\mathcal{H}_1)}}_{\text{prior odds}} \times \underbrace{\frac{p(D|\mathcal{H}_0)}{p(D|\mathcal{H}_1)}}_{\text{Bayes factor, } BF_{01}} = \underbrace{\frac{p(\mathcal{H}_0|D)}{p(\mathcal{H}_1|D)}}_{\text{posterior odds}}.$$

Bayes factors are *not* posterior odds — *Explanation*

$$\underbrace{\frac{p(\mathcal{H}_0)}{p(\mathcal{H}_1)}}_{\text{prior odds}} \times \underbrace{\frac{p(D|\mathcal{H}_0)}{p(D|\mathcal{H}_1)}}_{\text{Bayes factor, } BF_{01}} = \underbrace{\frac{p(\mathcal{H}_0|D)}{p(\mathcal{H}_1|D)}}_{\text{posterior odds}}.$$

Say that $BF_{01} = 32$; what does this mean?

> *After looking at the data, we revise our belief towards $\mathcal{H}_0$ by 32 times.*

Bayes factors are *not* posterior odds — *Explanation*

$$\underbrace{\frac{p(\mathcal{H}_0)}{p(\mathcal{H}_1)}}_{\text{prior odds}} \times \underbrace{\frac{p(D|\mathcal{H}_0)}{p(D|\mathcal{H}_1)}}_{\text{Bayes factor, } BF_{01}} = \underbrace{\frac{p(\mathcal{H}_0|D)}{p(\mathcal{H}_1|D)}}_{\text{posterior odds}}.$$

Say that $BF_{01} = 32$; what does this mean?

> *After looking at the data, we revise our belief towards $\mathcal{H}_0$ by 32 times.*

Q: What does this imply concerning the probability of each model, given the observed data?
A: On its own, nothing at all!

Bayes factors are *not* posterior odds — *Explanation*

$$\underbrace{\frac{p(\mathcal{H}_0)}{p(\mathcal{H}_1)}}_{\text{prior odds}} \times \underbrace{\frac{p(D|\mathcal{H}_0)}{p(D|\mathcal{H}_1)}}_{\text{Bayes factor, } BF_{01}} = \underbrace{\frac{p(\mathcal{H}_0|D)}{p(\mathcal{H}_1|D)}}_{\text{posterior odds}}.$$

Say that $BF_{01} = 32$; what does this mean?

> *After looking at the data, we revise our belief towards $\mathcal{H}_0$ by 32 times.*

Q: What does this imply concerning the probability of each model, given the observed data?
A: On its own, nothing at all!

Bayes factors $=$ rate of *change* of belief, not the *updated* belief.[1]

[1] Edwards, Lindman, and Savage (1963)

Bayes factors are *not* posterior odds — *What we found...*

> *"The alternative hypothesis is 2 times more likely than the null hypothesis ($B_{+0} = 2.46$; Bayesian 95% CI [0.106, 0.896])."*

Incidence:
- 13.2% as definition
- 20.4% as interpretation

Possible explanations:

- Principle of indifference.
- Overselling Bayes as the *theory of inverse probability.*[1]
- Cognitive dissonance.

[1] Jeffreys(1961)

4. Bayes factors — In applied research

Bayes factors are (at least can be) *sensitive* to priors

Bayes factors are (at least can be) *sensitive* to priors — *Explanation*

Very well known.[1,2,3,4,5]

$$\boxed{P(D|\mathcal{H}_i) = \int_{\Theta_i} p(D|\theta, \mathcal{H}_i) p(\theta|\mathcal{H}_i) d\theta}$$

Bayes factors are (at least can be) *sensitive* to priors — *Explanation*

Very well known.[1,2,3,4,5]

$$\boxed{P(D|\mathcal{H}_i) = \int_{\Theta_i} p(D|\theta, \mathcal{H}_i) p(\theta|\mathcal{H}_i) d\theta}$$

Example: Bias of a coin[6]

- $\mathcal{H}_0 : \theta = .5$ *vs* $\mathcal{H}_1 : \theta \neq .5$
- Data: 60 successes in 100 throws.
- Four within-model priors; all $Beta(a, b)$.

Prior	BF_{10}	Lee & Wagenmakers (2014)
Approx. to Haldane's prior ($a = .05$, $b = .05$)	0.09	'Strong' evidence for $\mathcal{H}_0$
Jeffreys' prior ($a = .5$, $b = .5$)	0.60	'Anecdotal' evidence for $\mathcal{H}_0$
Uniform prior ($a = 1$, $b = 1$)	0.91	'Anecdotal' evidence for $\mathcal{H}_0$
An informative prior ($a = 3$, $b = 2$)	1.55	'Anecdotal' evidence for $\mathcal{H}_1$

[1] Kass (1993) [2] Gallistel (2009) [3] Vanpaemel (2010) [4] Robert (2016) [5] Withers (2002) [6] Liu and Aitkin (2008)

Bayes factors are (at least can be) *sensitive* to priors — *What we found...*

Reporting nothing at all (29.9%) or relying on software defaults (35.3%) was quite common.

Bayes factors are (at least can be) *sensitive* to priors — *What we found...*

Reporting nothing at all (29.9%) or relying on software defaults (35.3%) was quite common.

Possible explanations:

- Lack of awareness.
- Economic writing style.
- Default priors to...
  ... ease comparison, avoid specification, meet 'objectivity'.
  Also: improve peer-review chances, principle of indifference, preregistration.

4. Bayes factors — In applied research

Bayes factors are a measure of *relative* evidence

Bayes factors are a measure of *relative* evidence — *Explanation*

Say that $BF_{01} = 100$; what does this mean?

> *The observed data are 100 times more likely under $\mathcal{H}_0$ than under this particular $\mathcal{H}_1$.*

Bayes factors are a measure of *relative* evidence — *Explanation*

Say that $BF_{01} = 100$; what does this mean?

> *The observed data are 100 times more likely under $\mathcal{H}_0$ than under this particular $\mathcal{H}_1$.*

- Evidence is *relative*.[1]
- A model may actually be dreadful, but simply less so than its competitor.[2,3]
- Little is known as to how Bayes factors behave under model misspecification (but see[4]).

[1] Morey, Romeijn, and Rouder (2016) [2] Rouder (2014) [3] Gelman and Rubin (1995) [4] Ly, Verhagen, and Wagenmakers (2016)

Bayes factors are a measure of *relative* evidence — *What we found...*

> *"With this 'stronger' VB05 prior, we found strong evidence for the null hypothesis (* $\mathrm{BFs}_{\mathrm{null}}$ *ranging from 12.7 to 22.7 for the 5 ROIs)."*

Incidence 62.3%

Possible explanations:

- Writing style.
- Implicitly assumed.
- Increased impact.

4. Bayes factors — In applied research

Bayes factors can *not* establish absence/presence

Bayes factors can *not* establish absence/presence — *Explanation*

Say that $BF_{01} = 100$, for $\mathcal{H}_0 : \mu = 0$ vs $\mathcal{H}_1 : \mu \neq 0$.

> *This does not imply that $\mu = 0$.*

Bayes factors can *not* establish absence/presence — *Explanation*

Say that $BF_{01} = 100$, for $\mathcal{H}_0 : \mu = 0$ vs $\mathcal{H}_1 : \mu \neq 0$.

> *This does not imply that* $\mu = 0$.

- First of all, the Bayes factor (as the p-value) is a stochastic endeavor, not a factual proof.
- Furthermore, the Bayes factor provides a relative assessment of the likelihood of the observed data, not of the entertained hypotheses.

Bayes factors can *not* establish absence/presence — *What we found...*

> *"For 6-year-olds, there was no difference between environments ($M_{smooth} = 2.11$ vs. $M_{rough} = 1.93$, $t(52) = 1.0, p = 0.31, d = 0.3, BF = .42$)."*

Incidence 35.3%

Possible explanations:

- Increased impact.
- Avoid uncertainty.
- Writing style.
- Influence from NHST.
- Decision making.

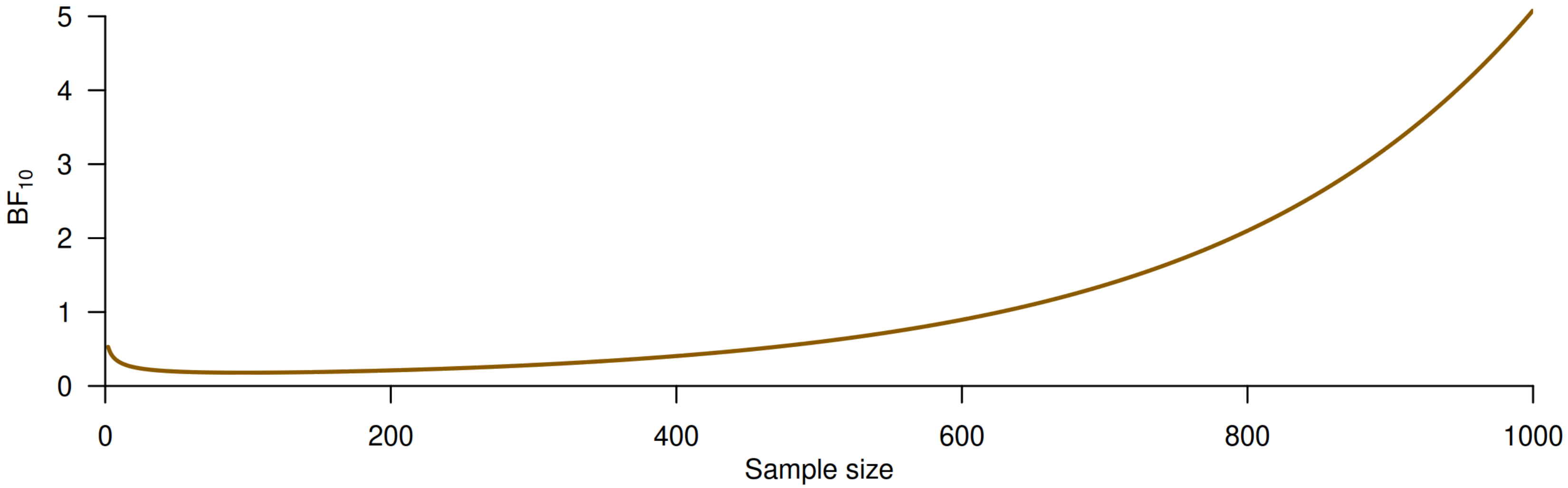
4. Bayes factors — In applied research

Bayes factors are *not* an effect size measure

Bayes factors are *not* an effect size measure — *Explanation*

Example:

- Bayesian one sample t-test: $\mathcal{H}_0 : \mu = 0$ vs $\mathcal{H}_1 : \mu \neq 0$.
- JZS default prior ($r = .707$).
- $\overline{x} = 0.1$, $sd = 1$ at each sample size (thus, the effect size is fixed throughout).



Bayes factors are *not* an effect size measure — *What we found...*

> *"Pupil size was larger in a higher tracking load (...). However, the Bayesian test showed only positive, but smaller, effect of Load on tracking pupil size ($BF_{\text{incl.}} = 7.506$)."*

Incidence 4.2%

Possible explanations:

- Recreating a similar misconception based on p-values.
- Bayes factor labels in use.

4. Bayes factors — In applied research

Inconclusive evidence is *not* evidence of absence

Inconclusive evidence is *not* evidence of absence — *Explanation*

$$\boxed{BF_{01} = \frac{p(D|\mathcal{H}_0)}{p(D|\mathcal{H}_1)} = 1}$$

> *Data are equally likely under either model.*

Inconclusive evidence is *not* evidence of absence — *Explanation*

$$\boxed{BF_{01} = \frac{p(D|\mathcal{H}_0)}{p(D|\mathcal{H}_1)} = 1}$$

> *Data are equally likely under either model.*

Data are perfectly uninformative.

This does not equate to "*there is nothing to be found*".

Inconclusive evidence is *not* evidence of absence — *What we found...*

> *"In contrast there was no difference in meaning between the thinking without examples and planning conditions; the Bayes factor provided anecdotal evidence in favor of the null ($BF_{10} = .86$)."*

Incidence 3.6%

Possible explanations:

- Recreating a similar misconception based on p-values.
- Absence as default.
- Dichotomization.
- Increased impact.
- Preference for parsimony.

4. Bayes factors — In applied research

Bayes factors are a *continuous* measure of relative evidence

Bayes factors are a *continuous* measure of relative evidence — *Explanation*

Bayes factors are a continuous measure of evidence in $[0, \infty)$.
For instance, if $BF_{01} > 1$ then

- The observed data are more likely under $\mathcal{H}_0$ than under $\mathcal{H}_1$.
- The larger BF_{01}, the stronger the evidence for $\mathcal{H}_0$ over $\mathcal{H}_1$.

Bayes factors are a *continuous* measure of relative evidence — *Explanation*

Bayes factors are a continuous measure of evidence in $[0, \infty)$.
For instance, if $BF_{01} > 1$ then

- The observed data are more likely under $\mathcal{H}_0$ than under $\mathcal{H}_1$.
- The larger BF_{01}, the stronger the evidence for $\mathcal{H}_0$ over $\mathcal{H}_1$.

Q: Can "*more likely than*" be qualified?
A: Several categorizations of strength of evidence (what is weak?, moderate?, strong?) exist.[1,2,3,4]

But this is problematic in various ways.

[1]Jeffreys (1961) [2]Kass and Raftery (1995) [3]Lee and Wagenmakers (2013) [4]Dienes (2016)

Bayes factors are a *continuous* measure of relative evidence — *What we found...*

> *"(...) In terms of Bayes factor (BF), evidence for greater disgust in the experimental group was strong ($BF_{10} > 10$), but there was only weak evidence for a difference in other emotions (BF_{10}'s < 3)."*

Incidence 5.4%

Possible explanations:

- Summary.
- Seeking authority.
- Avoiding criticism.
- Borrowing from the literature and JASP.
- NHST ('significant', 'not significant').

5. Conclusions, next steps

Conclusions (1/2)

I think that, concerning testing:

- Model comparison (including hypothesis testing) is really important.
- However, and clearly, researchers test *way* too much.
- Testing says very little about how well a model fits to data.

Conclusions (2/2)

And what about estimation?

I think that:

- Testing need not be a prerequisite for estimation, unlike what some advocate.[1]
- Estimation quantifies uncertainty in ways that Bayes factors simply can not.
- Estimating effect sizes (direction, magnitude) is crucial. Bayes factors ignore this!
- Avoiding the dichotomous reasoning subjacent to Bayes factors can help.

Bayes factors can be very useful (I use them!). But they should not always be the end of our inference.

[1]Wagenmakers et al. (2018)

What's next?

A follow-up study is in preparation.

- Create and deploy a Shiny app that illustrates correct and incorrect usage of the Bayes factor.
- Assess the efficacy of this app by means of an experiment.

Questions?